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A model for trimaximal lepton mixing

W. Grimus

University of Vienna, Faculty of Physics,
Boltzmanngasse 5, A-1090 Vienna, Austria
E-mail: walter.grimus@univie.ac.at

L. Lavoura

Universidade Técnica de Lisboa and Centro de Física Teórica de Partículas,
Instituto Superior Técnico, 1049-001 Lisboa, Portugal
E-mail: balio@cftp.ist.utl.pt

Abstract: We consider trimaximal lepton mixing, defined by $|U_{\alpha 2}|^2 = 1/3 \ \forall \alpha = e, \mu, \tau$. This corresponds to a two-parameter lepton mixing matrix U. We present a model for the lepton sector in which trimaximal mixing is enforced by softly broken discrete symmetries; one version of the model is based on the group $\Delta(27)$. A salient feature of our model is that no vacuum alignment is required.

Keywords: Neutrino Physics, Discrete and Finite Symmetries.

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Contents

1. Introduction

It is now experimentally firmly established that the neutrinos are massive and that leptons of different families mix in the charged weak interaction — see [1] for reviews and [2, 3] for recent fits. Lepton mixing is given by a 3×3 unitary matrix $(U_{\alpha j})$, with $\alpha = e, \mu, \tau$ corresponding to the lepton flavours and $j = 1, 2, 3$ corresponding to the neutrino mass eigenstates. According to the most recent three-flavour neutrino oscillation update [3], at the $3\,\sigma$ level, the results for neutrino mixing are

$$|U_{e3}|^2 \leq 0.056, \qquad 0.25 \leq \sin^2 \theta_\odot \leq 0.37, \qquad 0.36 \leq \sin^2 \theta_{\mathrm{atm}} \leq 0.67, \tag{1.1}$$

where

$$\sin^2 \theta_\odot \equiv \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \tag{1.2}$$

$$\sin^2 \theta_{\mathrm{atm}} \equiv \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}. \tag{1.3}$$

Moreover, at the $3\,\sigma$ level the neutrino mass differences satisfy [3]

$$7.05 \times 10^{-5}\,\mathrm{eV}^2 \leq \Delta m^2_\odot \equiv m_2^2 - m_1^2 \leq 8.34 \times 10^{-5}\,\mathrm{eV}^2, \tag{1.4}$$

$$2.07 \times 10^{-3}\,\mathrm{eV}^2 \leq \Delta m^2_{\mathrm{atm}} \equiv \left| m_3^2 - m_1^2 \right| \leq 2.75 \times 10^{-3}\,\mathrm{eV}^2, \tag{1.5}$$

the sign of $m_3^2 - m_1^2$ being unknown.

The bounds (1.1) suggest that the lepton mixing might be tri-bimaximal, i.e. that the lepton mixing matrix might be, apart from unphysical rephasings and from possible Majorana phases,

$$U = U_{\mathrm{HPS}} \equiv \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \tag{1.6}$$

The phenomenological hypothesis $U = U_{\mathrm{HPS}}$ has been put forward by Harrison, Perkins and Scott (HPS) [4]. However, it turns out that, at the model-building level, it is quite awkward to enforce $U = U_{\mathrm{HPS}}$ through symmetries. In particular, all existing models for $U = U_{\mathrm{HPS}}$ involve some form of vacuum alignment, i.e. two subsectors of the scalar sector having vacuum expectation values (VEVs) aligned in different directions; some papers where this problem has been discussed are found in [5], for more papers see the vast bibliography of [6].[1]

In this paper we adopt the milder hypothesis that lepton mixing is trimaximal, i.e. that

$$|U_{e2}|^2 = |U_{\mu 2}|^2 = |U_{\tau 2}|^2 = \frac{1}{3}. \tag{1.7}$$

Trimaximal mixing relaxes some of the HPS assumptions [4], since it allows for a nonzero U_{e3} as well as for $\sin^2\theta_{\mathrm{atm}} \neq 1/2$. Our main purpose in this paper is to show that trimaximal lepton mixing may be enforced through a simple model which involves no vacuum alignment.

In section 2 we make a brief phenomenological study of trimaximal mixing, proceeding in section 3 to present the simplest version of our model. In section 4 we consider some variations on the model. Our conclusions are found in section 5.

2. Trimaximal mixing

It follows from the trimaximal-mixing assumption $|U_{e2}|^2 = 1/3$ and equation (1.2) that

$$\sin^2\theta_\odot = \frac{1}{3\left(1 - |U_{e3}|^2\right)} \geq \frac{1}{3}, \tag{2.1}$$

which is somewhat disfavoured experimentally, since the best-fit value for $\sin^2\theta_\odot$ is $0.304 < 1/3$ [3]. The situation becomes worse for the trimaximal-mixing hypothesis when U_{e3} is nonzero; indeed, a recent fit [2] found $|U_{e3}|^2 = 0.016 \pm 0.010$, in agreement with $|U_{e3}|^2 = 0.01\,^{+0.016}_{-0.011}$ in [3], which is not yet a significant indication for a nonzero U_{e3}. In any case, the trimaximal-mixing hypothesis might be testable soon through more accurate measurements of $|U_{e2}|$ and $|U_{e3}|$.

A trimaximal lepton mixing matrix U has the moduli of two of its matrix elements *of the same column* fixed. This means that only two parameters remain in U,[2] which can be taken as $|U_{e3}|$, or the mixing angle θ_{13} and a Dirac phase. Clearly, for the latter a

[1] An alternative approach consists in using extra dimensions for model building, see for instance [7].

[2] Besides, two Majorana phases are present in U if the neutrinos are Majorana fermions.

convention has to be adopted. Using the convention for the Dirac phase δ promulgated by [8], we find

$$\tan 2\theta_{\mathrm{atm}} = \frac{1 - 2\left|U_{e3}\right|^2}{\left|U_{e3}\right| \cos\delta \sqrt{2 - 3\left|U_{e3}\right|^2}}. \tag{2.2}$$

In the following, we shall employ the parameterization of a trimaximal mixing matrix:

$$U = \mathrm{diag}\left(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}\right) U_{\mathrm{HPS}} \begin{pmatrix} c & 0 & se^{-i\psi} \\ 0 & 1 & 0 \\ -se^{i\psi} & 0 & c \end{pmatrix} \mathrm{diag}\left(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}\right), \tag{2.3}$$

where $c = \cos\theta$ and $s = \sin\theta$. The mixing angle θ parameterizes how much lepton mixing deviates from tri-bimaximality. The phase ψ is of Dirac type.[3] The phases $\delta_{e,\mu,\tau}$, together with one of the phases β_j, are unphysical; only the phase differences $2(\beta_1 - \beta_2)$ and $2(\beta_2 - \beta_3)$ can be physical, if the neutrinos happen to be of Majorana type. The modification (2.3) of the HPS mixing matrix has recently also been considered in [9].

From equations (2.3) and (1.6),

$$\left|U_{e3}\right|^2 = \frac{2}{3}\, s^2, \tag{2.4}$$

$$\sin^2\theta_{\mathrm{atm}} = \frac{1}{2} + \frac{cs\cos\psi}{\sqrt{3}\left(1 - \left|U_{e3}\right|^2\right)}. \tag{2.5}$$

Therefore,

$$\left(\sin^2\theta_{\mathrm{atm}} - \frac{1}{2}\right)^2 \le \frac{\left|U_{e3}\right|^2}{2}\, \frac{1 - \frac{3}{2}\left|U_{e3}\right|^2}{\left(1 - \left|U_{e3}\right|^2\right)^2}. \tag{2.6}$$

This inequality can also be obtained directly from equation (2.2). The inequality (2.6) relates, when the mixing is trimaximal, the maximal possible departure from maximal atmospheric-neutrino mixing, i.e. from $\sin^2\theta_{\mathrm{atm}} = 1/2$, to the value of $\left|U_{e3}\right|$.

3. A simple model

Introduction. Let us assume that the neutrinos are Majorana fermions. Then, in the weak basis in which the charged-lepton mass matrix is diagonal, the effective mass Lagrangian for the light neutrinos is

$$\mathcal{L}_{\text{neutrino mass}} = \frac{1}{2}\left(\nu_{eL}^T,\, \nu_{\mu L}^T,\, \nu_{\tau L}^T\right) C^{-1} \mathcal{M}_\nu \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} + \text{H.c.}, \tag{3.1}$$

where C is the Dirac-Pauli charge-conjugation matrix in Dirac space and $\mathcal{M}_\nu$ is a 3×3 symmetric matrix in flavour space. The lepton mixing matrix U diagonalizes $\mathcal{M}_\nu$:

$$U^T \mathcal{M}_\nu U = \mathrm{diag}\left(m_1, m_2, m_3\right), \tag{3.2}$$

[3] Note that the phase ψ corresponds to a Dirac phase convention different from that of δ.

the neutrino masses $m_{1,2,3}$ being non-negative real. Using the parameterization of a trimaximal U in equation (2.3), we shall denote

$$\mu_j \equiv m_j e^{-2i\beta_j} \quad \text{for } j = 1, 2, 3. \tag{3.3}$$

Then, if we assume the phases $\delta_{e,\mu,\tau}$ to vanish, we have

$$\mathrm{diag}\,(\mu_1, \mu_2, \mu_3) = \begin{pmatrix} c & 0 & -se^{i\psi} \\ 0 & 1 & 0 \\ se^{-i\psi} & 0 & c \end{pmatrix} \left(U_{\mathrm{HPS}}^T \mathcal{M}_\nu U_{\mathrm{HPS}} \right) \begin{pmatrix} c & 0 & se^{-i\psi} \\ 0 & 1 & 0 \\ -se^{i\psi} & 0 & c \end{pmatrix}. \tag{3.4}$$

Thus, up to the phase transformation given by the phases $\delta_{e,\mu,\tau}$, trimaximal mixing means that the vector $(1,1,1)^T$ is an eigenvector of $\mathcal{M}_\nu$ with eigenvalue μ_2. This means that, in the phase convention $\delta_e = \delta_\mu = \delta_\tau = 0$ for $\mathcal{M}_\nu$, the sum of the matrix elements of $\mathcal{M}_\nu$ over all rows and columns of $\mathcal{M}_\nu$ is equal (to μ_2). It is the purpose of this section to construct a model based on this idea.

The group $\boldsymbol{\Delta(27)}$. $\Delta(27)$ is a discrete group with 27 elements. It has two inequivalent triplet irreducible representations (irreps), $\underline{3}$ and $\underline{3}^*$, and nine inequivalent singlet irreps, $\underline{1}^{(p,q)}$ $(p, q = 0, 1, 2)$. The triplet irreps of $\Delta(27)$ are faithful, the singlet irreps are non-faithful. The group $\Delta(27)$ is generated by two transformations, C and T. In the $\underline{3}$, those transformations are represented as

$$\underline{3}: \quad C \to \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad T \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \text{with } \omega \equiv e^{2\pi i/3} = \frac{-1 + i\sqrt{3}}{2}. \tag{3.5}$$

Notice that the matrices representing C and T belong to SU(3), therefore $\Delta(27)$ may be viewed as a subgroup of SU(3). In the $\underline{3}^*$,

$$\underline{3}^*: \quad C \to \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad T \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}. \tag{3.6}$$

In the singlet irreps,

$$\underline{1}^{(p,q)}: \quad C \to \omega^p, \quad T \to \omega^q. \tag{3.7}$$

The relevant tensor products of irreps of $\Delta(27)$ are

$$\underline{3} \otimes \underline{3} = \underline{3}^* \oplus \underline{3}^* \oplus \underline{3}^*, \tag{3.8}$$

$$\underline{3} \otimes \underline{3}^* = \oplus_{p,q=0}^{2}\, \underline{1}^{(p,q)}. \tag{3.9}$$

Multiplets and symmetries. In our model we consider only the lepton sector and the electroweak interactions. The gauge group is the standard $\mathrm{SU}(2) \times \mathrm{U}(1)$. There are three left-handed-lepton doublets $D_{\alpha L} = (\nu_{\alpha L}, \alpha_L)^T$ and three right-handed charged-lepton singlets α_R $(\alpha = e, \mu, \tau)$. We add to these standard multiplets four right-handed neutrino singlets in order to enable the seesaw mechanism [10]. Those four right-handed neutrinos are divided in two sets, three $\nu_{\alpha R}$ and one ν_{0R}. In the scalar sector, there are four Higgs doublets, once again divided in two sets: three ϕ_α and one ϕ_0. We need moreover a scalar gauge singlet S. The $\mathrm{SU}(2) \times \mathrm{U}(1)$ and $\Delta(27)$ assignments of all these multiplets are given in table 1.

	$D_{\alpha L}$	α_R	$\nu_{\alpha R}$	ν_{0R}	ϕ_α	ϕ_0	S
$\mathrm{SU}(2)\times\mathrm{U}(1)$	$(\underline{2},-1)$	$(\underline{1},-2)$	$(\underline{1},0)$	$(\underline{1},0)$	$(\underline{2},1)$	$(\underline{2},1)$	$(\underline{1},0)$
$\Delta(27)$	$\underline{3}$	$\underline{3}^*$	$\underline{3}$	$\underline{1}^{(1,0)}$	$\underline{3}^*$	$\underline{1}^{(0,0)}$	$\underline{1}^{(1,0)}$

Table 1: Fermion and scalar multiplets of our model.

Additional $\mathbb{Z}_2$ symmetries. Besides the gauge group and $\Delta(27)$, we impose three extra $\mathbb{Z}_2$ symmetries $\mathbf{z}_{e,\mu,\tau}$:

$$\mathbf{z}_\alpha: \quad \alpha_R \to -\alpha_R,\ \phi_\alpha \to -\phi_\alpha, \tag{3.10}$$

while all other multiplets remain unchanged. Each $\mathbf{z}_\alpha$ has the purpose of "gluing" α_R to ϕ_α in the Yukawa couplings; this is the same idea as in [11] (see also [12]). Since the $\mathbf{z}_\alpha$ do not commute with $\Delta(27)$, the horizontal symmetry group employed in our model is actually much larger than $\Delta(27)$.

Yukawa couplings. Let us first consider the Yukawa couplings of the α_R. They are

$$\mathcal{L}_{\alpha_R\,\mathrm{Yukawas}} = -y_1 \sum_{\alpha=e,\mu,\tau} \bar{D}_{\alpha L}\alpha_R\phi_\alpha + \mathrm{H.c.} \tag{3.11}$$

According to equation (3.8), $\Delta(27)$ would allow two other couplings,

$$\begin{aligned} &-y_2\left(\bar{D}_{eL}\mu_R\phi_\tau + \bar{D}_{\mu L}\tau_R\phi_e + \bar{D}_{\tau L}e_R\phi_\mu\right) \\ &-y_3\left(\bar{D}_{eL}\tau_R\phi_\mu + \bar{D}_{\mu L}e_R\phi_\tau + \bar{D}_{\tau L}\mu_R\phi_e\right) + \mathrm{H.c.} \end{aligned} \tag{3.12}$$

These terms would destroy trimaximal mixing. It is for this reason that we have introduced into our model the symmetries $\mathbf{z}_\alpha$, which remove the terms (3.12) from the Lagrangian. The masses of the charged leptons are $m_\alpha = |y_1 v_\alpha|$, where v_α is the VEV of the neutral component of ϕ_α. If we manage $v_{e,\mu,\tau}$ to be all different, then the charged leptons will be non-degenerate in mass as desired.

The Yukawa couplings of the right-handed neutrinos are given by

$$\mathcal{L}_{\nu_R\,\mathrm{Yukawas}} = -y_4 \sum_{\alpha=e,\mu,\tau} \bar{D}_{\alpha L}\nu_{\alpha R}\left(i\tau_2\phi_0^*\right) + \frac{y_5}{2}\,\nu_{0R}^T C^{-1}\nu_{0R}\,S + \mathrm{H.c.} \tag{3.13}$$

Soft breaking of the symmetries. Soft breaking of (super)symmetries plays an important role in many models. Soft breaking is usually needed in models which want to explain mixing features through some symmetries. It has been emphasized that successful models need a residual symmetry [13]; for instance, in [11, 14] the residual symmetry after soft breaking is the μ–τ interchange symmetry, which leads to maximal atmospheric-neutrino mixing and to $U_{e3}=0$.

In the present model, we break $\Delta(27)$ softly in two steps. Firstly we allow it to be broken, by terms of dimension three, down to the $\mathbb{Z}_3$ symmetry generated by the transformation C, which is denoted $\mathbb{Z}_3(C)$. Secondly we allow $\mathbb{Z}_3(C)$ to be softly broken by terms of dimension two. We thus have the soft-breaking chain

$$\Delta(27) \stackrel{\dim 3}{\longrightarrow} \mathbb{Z}_3(C) \stackrel{\dim 2}{\longrightarrow} \{e\}, \tag{3.14}$$

where $\{e\}$ symbolizes the trivial group consisting only of the unit element.

The soft-breaking terms of dimension three occur in the Lagrangian of bare Majorana masses

$$\begin{aligned}
\mathcal{L}_{\text{Majorana masses}} = & \frac{M_0^*}{2} \sum_{\alpha=e,\mu,\tau} \nu_{\alpha R}^T C^{-1} \nu_{\alpha R} \\
& + M_1^* \left(\nu_{eR}^T C^{-1} \nu_{\mu R} + \nu_{\mu R}^T C^{-1} \nu_{\tau R} + \nu_{\tau R}^T C^{-1} \nu_{eR} \right) \\
& + \frac{M_2^*}{2} \left(\nu_{eR}^T + \omega\, \nu_{\mu R}^T + \omega^2\, \nu_{\tau R}^T \right) C^{-1} \nu_{0R} + \text{H.c.} \qquad (3.15)
\end{aligned}$$

The soft-breaking terms of dimension two occur in the scalar potential

$$V = c_e \phi_e^\dagger \phi_e + c_\mu \phi_\mu^\dagger \phi_\mu + c_\tau \phi_\tau^\dagger \phi_\tau + \cdots\,, \qquad (3.16)$$

the coefficients (with dimension mass squared) c_e, c_μ and c_τ being all different, thereby breaking $\mathbb{Z}_3(C)$. This is needed in order to obtain, upon spontaneous symmetry breaking, different VEVs $v_{e,\mu,\tau}$ and, therefore, different charged-lepton masses:

$$m_e : m_\mu : m_\tau = |v_e| : |v_\mu| : |v_\tau|\,. \qquad (3.17)$$

Furthermore, we might include in V all terms like $\phi_e^\dagger \phi_\mu$, etc. This would also break the symmetries $\mathbf{z}_\alpha$ softly and would avoid all potential problems with spontaneous breaking of discrete symmetries in our model.

Seesaw mechanism. From equations (3.13) and (3.15), we see that there are in our model Majorana and Dirac neutrino mass matrices

$$M_R = \begin{pmatrix} M_0 & M_1 & M_1 & M_2 \\ M_1 & M_0 & M_1 & \omega^2 M_2 \\ M_1 & M_1 & M_0 & \omega M_2 \\ M_2 & \omega^2 M_2 & \omega M_2 & M_N \end{pmatrix}, \quad M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}, \qquad (3.18)$$

respectively. We have defined $M_N = y_5^* v_S^*$, where v_S is the VEV of the scalar singlet S, and $a = y_4^* v_0$, where v_0 is the VEV of the neutral component of ϕ_0. The seesaw mechanism [10] tells us that

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D. \qquad (3.19)$$

After some algebra one finds that $\mathcal{M}_\nu$ is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} x+y & z+\omega^2 y & z+\omega y \\ z+\omega^2 y & x+\omega y & z+y \\ z+\omega y & z+y & x+\omega^2 y \end{pmatrix}, \qquad (3.20)$$

where

$$x = -a^2\, \frac{M_0 + M_1}{(M_0 - M_1)(M_0 + 2M_1)}\,, \qquad (3.21)$$

$$z = a^2\, \frac{M_1}{(M_0 - M_1)(M_0 + 2M_1)}\,, \qquad (3.22)$$

$$y = -a^2\, \frac{M_2^2}{M_N (M_0 - M_1)^2}\,. \qquad (3.23)$$

It is clear that

$$\mathcal{M}_\nu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (x+2z) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{3.24}$$

Therefore, our model predicts trimaximal mixing.

A further prediction. We compute

$$U_{\mathrm{HPS}}^T \mathcal{M}_\nu U_{\mathrm{HPS}} = \begin{pmatrix} x - z + \frac{3}{2}\, y & 0 & i\, \frac{3}{2}\, y \\ 0 & x + 2z & 0 \\ i\, \frac{3}{2}\, y & 0 & x - z - \frac{3}{2}\, y \end{pmatrix}. \tag{3.25}$$

Comparing this result with equation (3.4), we see that

$$\mu_2 = x + 2z = \frac{-a^2}{M_0 + 2M_1} \tag{3.26}$$

and

$$x - z + \frac{3}{2}\, y = \mu_1 c^2 + \mu_3 s^2 e^{2i\psi}, \tag{3.27}$$

$$x - z - \frac{3}{2}\, y = \mu_3 c^2 + \mu_1 s^2 e^{-2i\psi}, \tag{3.28}$$

$$i\, \frac{3}{2}\, y = cs \left(\mu_3 e^{i\psi} - \mu_1 e^{-i\psi} \right). \tag{3.29}$$

Therefore,

$$\frac{\mu_1}{\mu_3} = \left(\frac{c - ise^{i\psi}}{c - ise^{-i\psi}} \right)^2, \tag{3.30}$$

hence

$$\frac{m_1}{m_3} = \frac{1 + 2cs \sin\psi}{1 - 2cs \sin\psi}. \tag{3.31}$$

Comparing this result with equations (2.4) and (2.5), one finds that

$$|U_{e3}|^2 \left(2 - 3\, |U_{e3}|^2 \right) - \left(1 - |U_{e3}|^2 \right)^2 \left(2 \sin^2 \theta_{\mathrm{atm}} - 1 \right)^2 = \frac{1}{3}\, \frac{\left(m_3^2 - m_1^2 \right)^2}{\left(m_1 + m_3 \right)^4}, \tag{3.32}$$

cf. inequality (2.6). The prediction (3.32) relates the deviation from tri-bimaximal lepton mixing to the mass ratio m_1/m_3. Using the experimental value of $\Delta m^2_{\mathrm{atm}}$, then, the sum of neutrino masses $m_1 + m_3$ is determined by the deviation from tri-bimaximal mixing.

With the experimental $3\,\sigma$ bounds (1.1) and (1.5), one finds

$$m_1 + m_3 \geq 0.060\, \mathrm{eV}. \tag{3.33}$$

If $|U_{e3}|^2$ is smaller than the bound (1.1) and/or if $\sin^2 \theta_{\mathrm{atm}}$ deviates from 1/2, then the lower bound (3.33) on $m_1 + m_3$ is strengthened. However, the fourth power of $m_1 + m_3$ in equation (3.32) dampens this effect.

Taking the experimental values of $\Delta m^2_{\mathrm{atm}}$ and $\Delta m^2_\odot$ as input, equation (3.32) determines the smallest neutrino mass m_{min}, which is m_1 for the normal and m_3 for the inverted

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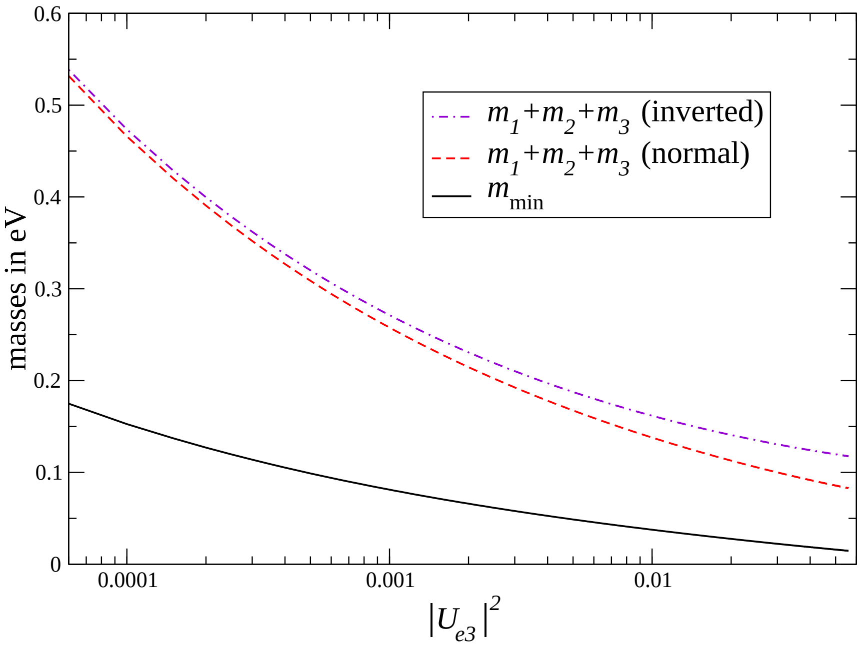

Figure 1: The minimal neutrino mass and the sum of the neutrino masses, for both types of spectra, as a function of $|U_{e3}|^2$. We have fixed the mass-squared differences at the mean values given in [3] and assumed that atmospheric mixing is maximal.

spectrum, as a function of $|U_{e3}|^2$ and $\sin^2\theta_{\mathrm{atm}}$. In figure 1 we have depicted m_{min} as a function of $|U_{e3}|^2$, fixing $\sin^2\theta_{\mathrm{atm}}$ at 0.5 and using the mean values $\Delta m^2_{\mathrm{atm}} = 2.4 \times 10^{-3}\,\mathrm{eV}^2$ and $\Delta m^2_{\odot} = 7.65 \times 10^{-5}\,\mathrm{eV}^2$ from [3]. We also show the sum of the neutrino masses for both the normal and the inverted spectra. We see that at large $|U_{e3}|^2$ the sum of the neutrino masses is safely below present cosmological bounds [15].

Parameter counting and the number of predictions. The neutrino mass matrix (3.20) is a five-parameter mass matrix because it has three complex parameters, with only their relative phases having a physical meaning. One can easily show with equation (3.25) that $\mathcal{M}_\nu^\dagger \mathcal{M}_\nu$ has only four parameters. The neutrino masses, the mixing angles and the Dirac phase follow all from $\mathcal{M}_\nu^\dagger \mathcal{M}_\nu$, only for the investigation of the Majorana phases we need in fact $\mathcal{M}_\nu$.

Therefore, the four parameters in $\mathcal{M}_\nu^\dagger \mathcal{M}_\nu$ determine seven observables. As a consequence, there must be three predictions. Indeed, those predictions are given by equations (2.1) and (2.2), which follow from trimaximal mixing alone, and equation (3.32), which is a result of our specific model.

As for $\mathcal{M}_\nu$, the difference of the two Majorana phases can be expressed in terms of its five parameters; this constitutes the additional prediction if we consider $\mathcal{M}_\nu$ instead of $\mathcal{M}_\nu^\dagger \mathcal{M}_\nu$. However, in our model, we expect no significant result for the effective neutrino mass in neutrinoless double β-decay as compared to the general case, since one Majorana phase is competely free and $|U_{e3}|^2$ is small.

4. Variations on the model

4.1 Use of a CP symmetry

The model presented in the previous section does not possess μ–τ interchange symmetry, which would require, in the $\mathcal{M}_\nu$ of equation (3.20), $y = 0$, leading to two degenerate neutrinos. Alternatively, though, we may impose on the model a CP symmetry which interchanges the μ and τ families [16], *viz.*

$$\begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} (t, \vec{r}) \xrightarrow{CP} i\,\gamma_0\, C \begin{pmatrix} \bar{\nu}_{eR}^T \\ \bar{\nu}_{\tau R}^T \\ \bar{\nu}_{\mu R}^T \end{pmatrix} (t, -\vec{r}) , \quad \nu_{0R}\,(t, \vec{r}) \xrightarrow{CP} i\,\gamma_0\, C\, \bar{\nu}_{0R}^T\,(t, -\vec{r}) , \tag{4.1}$$

and so on. Such a CP symmetry would force $M_{0,1,2}$ in equation (3.15) to be real, hence x, y and z in equations (3.20)–(3.23) to be real. One would thus obtain a neutrino mass matrix with three parameters only, which fulfils

$$S \mathcal{M}_\nu S = \mathcal{M}_\nu^* \quad \text{with} \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{4.2}$$

As shown in [16], a neutrino mass matrix obeying equation (4.2) predicts $\sin^2\theta_{\mathrm{atm}} = 1/2$, maximal CP violation, i.e. $e^{i\delta} = \pm i$, and vanishing Majorana phases. Therefore, a restricted version of our model, including the CP symmetry (4.1), has these predictions in addition to those of trimaximal mixing.

4.2 One more right-handed neutrino

If one wants to have trimaximal mixing without the extra prediction (3.32), one may introduce into the model one more right-handed neutrino, ν'_{0R}, transforming as $\underline{1}^{(2,0)}$ under $\Delta(27)$. This leads to one extra term

$$\frac{y_6}{2}\, \nu'^T_{0R} C^{-1} \nu'_{0R}\, S^* + \text{H.c.} \tag{4.3}$$

on the right-hand side of equation (3.13), and to two extra couplings

$$\frac{M_3^*}{2} \left(\nu_{eR}^T + \omega^2\, \nu_{\mu R}^T + \omega\, \nu_{\tau R}^T \right) C^{-1} \nu'_{0R} + M_4^* \nu_{0R}^T C^{-1} \nu'_{0R} + \text{H.c.} \tag{4.4}$$

on the right-hand side of equation (3.15). Equations (3.18) would then read

$$M_R = \begin{pmatrix} M_0 & M_1 & M_1 & M_2 & M_3 \\ M_1 & M_0 & M_1 & \omega^2 M_2 & \omega M_3 \\ M_1 & M_1 & M_0 & \omega M_2 & \omega^2 M_3 \\ M_2 & \omega^2 M_2 & \omega M_2 & M_N & M_4 \\ M_3 & \omega M_3 & \omega^2 M_3 & M_4 & M'_N \end{pmatrix} , \quad M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{4.5}$$

	$D_{L\alpha}$	α_R	$\nu_{\alpha R}$	ϕ_α
T	T	T^*	T	T^2

Table 2: Generalizing T.

with $m'_N = y_6^* v_S$. Instead of equation (3.20) one would then have

$$\mathcal{M}_\nu = \begin{pmatrix} x+y+t & z+\omega^2 y+\omega t & z+\omega y+\omega^2 t \\ z+\omega^2 y+\omega t & x+\omega y+\omega^2 t & z+y+t \\ z+\omega y+\omega^2 t & z+y+t & x+\omega^2 y+\omega t \end{pmatrix}. \tag{4.6}$$

This still predicts trimaximal mixing but the extra prediction (3.32) disappears.

4.3 Use of $\Delta(3n^2)$ or other symmetry groups

We may generalize the symmetry T by using, instead of equation (3.5),

$$\underline{3}: \quad C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma^* \end{pmatrix}, \quad \text{with } \sigma = e^{2\pi i/n} \quad (n \geq 3). \tag{4.7}$$

The transformation properties of the multiplets under T would then be as shown in table 2; fields not shown in that table transform trivially under T.

All fields transform under C in the same way as in section 3. Apart from T, all other details of the model are the same, in particular the soft breaking as given by equation (3.14). Thus, the matrix M_R would remain unchanged, since its form is fixed by the transformation C.

Let us consider $n \geq 4$. Then the main conclusions are the following:

- The fermionic sector and, therefore, the matrix $\mathcal{M}_\nu$ is the same for all n, as given in equation (3.18).
- The terms in equation (3.12) are automatically forbidden, therefore the $\mathbb{Z}_2$ symmetries of equation (3.10) are not needed.
- The symmetry group is $\Delta(3n^2)$, softly broken by terms of dimension three to $\mathbb{Z}_3(C)$.

A detailed discussion of $\Delta(3n^2)$ is given in [17]. Actually, the terms of dimension four in the Lagrangian are invariant under *all* the permutations. This leads to the symmetry group $\Delta(6n^2)$—see [17].

There is still another way to produce the present model. Consider cyclic permutations (or all permutations), plus family lepton-number symmetries $\mathrm{U}(1)_\alpha$ and the $\mathbb{Z}_2$ symmetries of equation (3.10). The scalar doublets carry no lepton number. The neutrino ν_{0R} and the scalar singlet S may either carry lepton number or not. The $\mathrm{U}(1)_\alpha$ are softly broken by terms of dimension three, the residual symmetry being once again $\mathbb{Z}_3(C)$.

All the groups considered here produce identical models as far as the terms in the Lagrangian involving the fermion fields are concerned; the only differences which may arise occur in the terms of dimension four in the scalar potential.

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5. Conclusions

In this paper we have focused our attention on trimaximal lepton mixing, with a two-parameter lepton mixing matrix, which generalizes tri-bimaximal mixing. Our main motivation was to allow for a deviation of $|U_{e3}|^2$ from zero; recent studies point out that possibility [2, 3]. Trimaximal mixing correlates the deviation of $|U_{e3}|^2$ from zero with a deviation of $\sin^2\theta_\odot$ from $1/3$ and of $\sin^2\theta_{\rm atm}$ from $1/2$ — see equations (2.1) and (2.2). A particular consequence is $\sin^2\theta_\odot \geq 1/3$, which is slightly disfavoured by the data at the moment, but in any case might be tested soon.

We have also constructed a seesaw model (or rather a class of models) where trimaximal mixing is enforced by a family symmetry group. In this model, the mass matrix of the light neutrinos, given by equation (3.20), has five physical parameters; it includes not only the predictions of trimaximal mixing but also equation (3.32) which relates the deviation from tri-bimaximal lepton mixing to the ratio m_1/m_3 of neutrino masses. As for a family symmetry group, we have considered several possibilities; one of the most straightforward ones is based on the group $\Delta(27)$. We have also considered a restricted version of our model by imposing, in addition, a non-standard CP transformation; in this way we are lead to a three-parameter neutrino mass matrix which predicts not only trimaximal mixing but also $\sin^2\theta_{\rm atm} = 1/2$.

Our model has some peculiarities, like the need of four (or more) right-handed neutrino singlets; the fourth neutrino singlet couples to the the three usual ones denoted by $\nu_{\alpha R}$ ($\alpha = e, \mu, \tau$) via a scalar gauge singlet. In the mass terms of the $\nu_{\alpha R}$, the symmetry group is broken softly down to a $\mathbb{Z}_3$. An outstanding feature of the model is that no vacuum alignment is required, despite its scalar content of four Higgs doublets and the scalar singlet. This is to be contrasted with models for tri-bimaximal mixing which are plagued by the intricacies of vacuum alignment.

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